[] A *function* $f : \mathbb{X} \to \mathbb{Y}$ takes elements from $\mathbb{X}$ as the input and provides elements from $\mathbb{Y}$ as the output. It is important to note that

- f produces only one element from $\mathbb{Y}$ as the output for any $x \in \mathbb{X}$.
- It must produce an output for every $x \in \mathbb{X}$ otherwise it does not qualify as a function.
- $f(x) \in \mathbb{Y}$ is called the image of $x \in \mathbb{X}$.
- $x \in \mathbb{X}$ is called the pre-image of $f(x) \in \mathbb{Y}$.
- We may thus say that for each $x \in \mathbb{X}$ the function produces a *unique* image $f(x) \in \mathbb{Y}$.

[] A self-map $f : (\mathbb{X}, d_X) \to (\mathbb{X}, d_X)$ is a *contraction* if

- $\exists\ K \in (0, 1)$ such that
- $d(f(x), f(y)) \leq K d(x, y)$
- $\forall x, y \in \mathbb{X}$.

[] Given a non-empty set $\mathbb{X}$, let $\overline{P(\mathbb{X})}$ denote the collection of all non-empty subsets of $\mathbb{X}$.

[] A *correspondence* $f^c : \mathbb{X} \to \overline{P(\mathbb{Y})}$ takes elements from $\mathbb{X}$ as the input and provides elements from $\overline{P(\mathbb{Y})}$ as the output. It is important to note that

- f^c produces only one element from $\overline{P(\mathbb{Y})}$ as the output for any $x \in \mathbb{X}$; but one element of $\overline{P(\mathbb{Y})}$ may contain one or more elements of $\mathbb{Y}$.
- It must produce an output for every $x \in \mathbb{X}$ otherwise it does not qualify as a correspondence.
- $f^c(x) \in \overline{P(\mathbb{Y})}$ is called the image of $x \in \mathbb{X}$.
- $x \in \mathbb{X}$ is called the pre-image of $f^c(x) \in \overline{P(\mathbb{Y})}$.
- We will denote $f^c : \mathbb{X} \to \overline{P(\mathbb{Y})}$ as $f^c : \mathbb{X} \rightrightarrows \mathbb{Y}$ throughout this section.