1.4 CP Violation in the Standard Model

1.4.1 The CKM Picture of CP Violation

In the Standard Model (SM) [24] of $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge symmetry with three fermion generations, CP violation arises from a single phase in the mixing matrix for quarks [3]. Each quark generation consists of three multiplets:

$$Q_L^I = \begin{pmatrix} U_L^I \\ D_L^I \end{pmatrix} = (3,2)_{+1/6}, \quad u_R^I = (3,1)_{+2/3}, \quad d_R^I = (3,1)_{-1/3}, \tag{1.62}$$

where $(3,2)_{+1/6}$ denotes a triplet of $SU(3)_C$, doublet of $SU(2)_L$ with hypercharge $Y = Q \Leftrightarrow T_3 = +1/6$, and similarly for the other representations. The interactions of quarks with the $SU(2)_L$ gauge bosons are given by

$$\mathcal{L}_W = \Leftrightarrow \frac{1}{2} g \overline{Q_{Li}^I} \gamma^\mu \tau^a \mathbf{1}_{ij} Q_{Lj}^I W_\mu^a, \tag{1.63}$$

where γ^μ operates in Lorentz space, τ^a operates in $SU(2)_L$ space and $\mathbf{1}$ is the unit matrix operating in generation (flavor) space. This unit matrix is written explicitly to make the transformation to mass eigenbasis clearer. The interactions of quarks with the single Higgs scalar doublet $\phi(1,2)_{+1/2}$ of the Standard Model are given by

$$\mathcal{L}_Y = \Leftrightarrow G_{ij} \overline{Q_{Li}^I} \phi d_{Rj}^I \Leftrightarrow F_{ij} \overline{Q_{Li}^I} \tilde{\phi} u_{Rj}^I + \text{Hermitian conjugate}, \tag{1.64}$$

where G and F are general *complex* 3×3 matrices. Their complex nature is the source of CP violation in the Standard Model. With the spontaneous symmetry breaking, $SU(2)_L \times U(1)_Y \to U(1)_{\text{EM}}$ due to $\langle \phi \rangle \neq 0$, the two components of the quark doublet become distinguishable, as are the three members of the W^μ triplet. The charged current interaction in (1.63) is given by

$$\mathcal{L}_W = \Leftrightarrow \sqrt{\frac{1}{2}} g \overline{u_{Li}^I} \gamma^\mu \mathbf{1}_{ij} d_{Lj}^I W_\mu^+ + \text{h.c.}. \tag{1.65}$$

The mass terms that arise from the replacement $\mathcal{R}e(\phi^0) \to \sqrt{\frac{1}{2}}(v + H^0)$ in (1.64) are given by

$$\mathcal{L}_M = \Leftrightarrow \sqrt{\frac{1}{2}} v G_{ij} \overline{d_{Li}^I} d_{Rj}^I \Leftrightarrow \sqrt{\frac{1}{2}} v F_{ij} \overline{u_{Li}^I} u_{Rj}^I + \text{Hermitian conjugate}, \tag{1.66}$$

namely

$$M_d = Gv/\sqrt{2}, \quad M_u = Fv/\sqrt{2}. \tag{1.67}$$

The phase information is now contained in these mass matrices. To transform to the mass eigenbasis, one defines four unitary matrices such that

$$V_{dL} M_d V_{dR}^\dagger = M_d^{\text{diag}}, \quad V_{uL} M_u V_{uR}^\dagger = M_u^{\text{diag}}, \tag{1.68}$$