

Answer: Have two independent normal distributions with unknown variances.

Wrens: $\bar{x}_1 = 21.18$ mm., $s_1^2 = 0.6418$, $n_1 = 10$.

Reed warblers: $\bar{x}_2 = 22.14$ mm., $s_2^2 = 0.4116$, $n_2 = 10$.

Assume $\sigma_1^2 = \sigma_2^2 = \sigma^2$ (unknown). Estimate σ^2 using

$$s^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2} = \frac{9s_1^2 + 9s_2^2}{18} = 0.5267.$$

Also $\bar{x}_1 - \bar{x}_2 = 21.18 - 22.14 = -0.96$, $\sqrt{s^2 \left(\frac{1}{n_1} + \frac{1}{n_2} \right)} = 0.1053$, $t_{18}(2.5\%) = 2.101$.

If $\mu_1 = \mu_2$ then the two groups of eggs have the same mean length.

To test $H_0 : \mu_1 = \mu_2$ vs. $H_1 : \mu_1 \neq \mu_2$ at 5% level, reject H_0 if $\left| \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s^2 (1/n_1 + 1/n_2)}} \right| \geq t_8(2.5\%)$.

Here $\left| \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s^2 (1/n_1 + 1/n_2)}} \right| = \left| \frac{-0.96}{\sqrt{0.1052}} \right| = 2.95$ so reject the null hypothesis of equal means at 5% level. The two groups of eggs are significantly different at 5% level.

This does not necessarily imply cuckoos can control their egg size. It has been proposed that a cuckoo lays its egg in the particular nest for which it is best adapted. For further information see: Wyllie, I. (1981) The Cuckoo. Batsford: London.

Davies, N.B. and Brooke, M. Coevolution of the cuckoo and its host, Scientific American, January 1991, p.66-73.