Question (lecture 15).
Two independent samples gave values 3, 6, 5, 2 for sample 1 and 2, 2, 3, 3, 5 for sample 2. Assuming that the samples come from independent normal distributions with common unknown variance σ^2, test at the 5% level whether the difference in mean equals zero against the alternative that it does not equal zero.
Answer: [31]

Question (lecture 15).
Five randomly selected remuneration packages for US oil and gas CEOs in 2008 were (in thousands of US dollars) 21333, 7294, 6712, 5727, 7087. Five randomly selected remuneration packages for US health care CEOs in 2008 were (in thousands of dollars) 14262, 8381, 7245, 10211, 1817. Test at the 5% level whether the difference in mean remuneration equals zero against the alternative hypothesis that it does not equal zero. You can assume that the two populations have common (unknown) variance σ^2.
Answer: [32]

Question (lecture 16).
A quarter of insurance claims are incomplete in some way. If you have 250 forms to process, what is the approximate probability that you will find fewer than 50 of them incomplete?
Answer: [33]

Question (lecture 16).
In $n = 100$ tosses of a coin I obtain $X = 72$ heads. Obtain an approximate 95% confidence interval for the probability θ of a head.
Answer: [34]

Question (lecture 17).
In December 2010 two analysts suggested several shares as likely to rise in 2011. By the end of October 2011 one (Neil Woodford) had four out of $n_1 = 7$ "share tips" showing a rise while the other (Harry Nummo) had three out of $n_2 = 10$ "share tips" showing a rise. Test at the 5% level whether the two success proportions are significantly different.
Answer: [35]

$z = \dfrac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \dfrac{4-3}{\sqrt{\frac{4}{4} + \frac{1}{5}}} = 0.913$. Test rule is reject H_0 if $|z| > 1.96$. Thus accept H_0 at 5% level.

[31] $n_1 = 4$, $\bar{x}_1 = 4$, $s_1^2 = 3.333$, $n_2 = 5$, $\bar{x}_2 = 3$, $s_2^2 = 1.5$, pooled estimate of σ^2 is $s^2 = \dfrac{3s_1^2 + 4s_2^2}{7} = 2.2857$. Testing H_0: $\mu_1 - \mu_2 = 0$ *vs.* H_1: $\mu_1 - \mu_2 \neq 0$. Test statistic is $t = \dfrac{\bar{x}_1 - \bar{x}_2}{s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \dfrac{4-3}{1.5119 \times \sqrt{\frac{1}{4} + \frac{1}{5}}} = 0.986$. Test rule is reject H_0 if $|t| > t_7(2.5\%)$. As $t_7(2.5\%) = 2.365$, accept H_0 at 5% level.

[32] Data source: `http://graphicsweb.wsj.com/php/CEOPAY09.html`.
$n_1 = 5$, $\bar{x}_1 = 9630.6$, $s_1^2 = 43158021$, $n_2 = 5$, $\bar{x}_2 = 8383.2$, $s_2^2 = 20577907$, $n_1 + n_2 - 2 = 8$, $t_8(2.5\%) = 2.306$. If variances are equal to σ^2, estimate σ^2 using $s^2 = \dfrac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1 + n_2 - 2} = 31867964$. Test statistic is $t = \frac{|\bar{x}_1 - \bar{x}_2|}{\sqrt{s^2(\frac{1}{n_1} + \frac{1}{n_2})}} = \frac{1247.4}{3570.32} = 0.349$. Since $t_8(2.5\%) = 2.306$, then $|t| < t_8(2.5\%)$ so accept H_0 that $\mu_1 = \mu_2$ against the alternative $\mu_1 \neq \mu_2$ at the 5% level.

[33] If X is the number of incomplete forms, $X \sim \text{Bin}(n = 250, \theta = \frac{1}{4}) \approx \text{N}(\mu = 62.5, \sigma^2 = 46.875)$. You require $\text{pr}\{X < 50\} = \text{pr}\{X \leq 49\} = \Phi\left(\dfrac{49 + \frac{1}{2} - \mu}{\sigma}\right) = \Phi(-1.899) = 0.0288$. Notice we have used a continuity correction.

[34] Number of heads $X \sim \text{Bin}(n = 100, \theta)$. Here $n = 100$, $X = 72$ observed, $\hat{\theta} = X/n = 72/100 = 0.72$. Approximate 95% confidence interval is $\hat{\theta} \pm 1.96\sqrt{\dfrac{\hat{\theta}(1-\hat{\theta})}{n}} = 0.72 \pm 0.088$.

[35] Data source: `http://www.thisismoney.co.uk/money/investing/article-1709914/Stock-market-predict`