Question (lecture 17).

In January 2011 Durham police were reported as disappointed by the increase in the number of people arrested for drinking and driving. Between December 1st 2010 and December 31st 2010 they had 52 positive breath tests out of 1799 breath tests administered, while for the same period in 2009 they had 41 positive tests out of 1433 administered. Construct a 95% confidence interval for the difference in proportion of drivers who tested positive. Source: `http://www.bbc.co.uk/news/uk-england-12261462`

Answer: [36]

Question (lecture 17).

I observe two dice. For one die I notice that it gives a six 20 times out of 100 and for the second die I notice that it gives a six 22 times out of 80. Test at the 5% level whether the two dice give the same probability of showing a six.

Answer: [37]

Question (lecture 18).

If $X \sim \chi^2_4$, for what value of x is $\text{pr}\{X > x\} = 0.05$?

Answer: [38]

Question (lecture 19).

I roll a die 100 times and observe the following results.

Outcome i	1	2	3	4	5	6
Observed frequency	16	15	16	15	15	23

Test at the 5% level whether the die is fair.

Answer: [39]

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Two binomial proportions here. $\hat\theta_1 = 4/7 = 0.571$, $\hat\theta_2 = 3/10 = 0.300$, $n_1 = 7$, $n_2 = 10$. Common estimated proportion is $\theta = \dfrac{7\hat\theta_1 + 10\hat\theta_2}{17} = 0.412$. Approximate test statistic is $z = \dfrac{|\hat\theta_1 - \hat\theta_2|}{\sqrt{\hat\theta(1-\hat\theta)\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} = 1.119$. reject H_0 at 5% level if $|z| > 1.96$, so here accept the hypothesis that the two proportions are equal.

[36] Two binomial proportions again. $\hat\theta_1 = 52/1799 = 0.028905$, $\hat\theta_2 = 41/1433 = 0.028611$, $n_1 = 1799$, $n_2 = 1433$.

Common estimated proportion is $\theta = \dfrac{1799\hat\theta_1 + 1433\hat\theta_2}{3232} = 0.0288$. (This is very small so the normal approximation is doubtful. In practice we would transform to give approximate normality.) Approximate test statistic is $z = \dfrac{|\hat\theta_1 - \hat\theta_2|}{\sqrt{\hat\theta(1-\hat\theta)\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} = 0.0496$. Reject H_0 at 5% level if $|z| > 1.96$, so here accept the hypothesis that the two proportions are equal.

[37] $n_1 = 100$, $x_1 = 20$, $\hat\theta_1 = 20/100 = 0.200$, $n_2 = 80$, $x_2 = 22$, $\hat\theta_2 = 22/80 = 0.275$. We test H_0: $\theta_1 = \theta_2 (= \theta)$ *vs.* H_1: $\theta_1 \neq \theta_2$. This is equivalent to testing H_0: $\theta_1 - \theta_2 = 0$ *vs.* H_1: $\theta_1 - \theta_2 \neq 0$. Assuming H_0 is true, the estimated common proportion θ is estimated by $\hat\theta = \dfrac{n_1\hat\theta_1 + n_2\hat\theta_2}{n_1+n_2} = \dfrac{20+22}{180} = 0.2333$. Test statistic is $z = \dfrac{\hat\theta_1 - \hat\theta_2}{\sqrt{\frac{\hat\theta(1-\hat\theta)}{n_1} + \frac{\hat\theta(1-\hat\theta)}{n_2}}} = \dfrac{0.200 - 0.275}{\sqrt{0.0017889 + 0.0014907}} = -1.31$. Test rule is reject H_0 if $|z| > 1.96$, so accept H_0 at 5% level.

[38] From tables, $x = 9.488$.

[39] Let X denote the outcome of the die. We test whether $\text{pr}\{X = i\} = 1/6$ for all i. Expected frequency for any outcome would then be $100 \times \frac{1}{6} = 16.667$.

Outcome i	1	2	3	4	5	6	
Observed frequency O_i	16	15	16	15	15	23	
Expected frequency E_i	16.67	16.67	16.67	16.67	16.67	16.67	
$(O_i - E_i)^2/E_i$	0.0267	0.1667	0.0267	0.1667	0.1667	2.407	sum=2.960