

By the way, even if you're just doing a quick calculation, I recommend **not** using a calculator. Enter the data into an Excel spreadsheet so that you can add/change/scrutinize/save calculations as needed. Sometimes I see an obviously invalid result and when I ask, "How did you get that result? What numbers did you use?" the answer is "I put the numbers into the calculator and this was the result I got." But how do you know you entered the numbers and formulas correctly? What if you need to re-do the calculation for a slightly different set of numbers?

Examples of the use of units and scrutiny

These examples, particularly the first one, also introduce the concept of "back of the envelope" estimates, a powerful engineering tool.

Example 1. Drag force and power requirements for an automobile

A car with good aerodynamics has a drag coefficient (C_D) of 0.2. The drag coefficient is defined as the ratio of the drag force (F_D) to the *dynamic pressure* of the flow = $\frac{1}{2}\rho v^2$ (where ρ is the fluid density and v the fluid velocity far from the object) multiplied by the cross-section area (A) of the object, *i.e.*

$$F_D = \frac{1}{2} C_D \rho v^2 A \quad \text{(Equation 7)}$$

The density of air at standard conditions is 1.18 kg/m^3 .

(a) Estimate the power required to overcome the aerodynamic drag of such a car at 60 miles per hour.

Power = Force x velocity

$$v = 60 \text{ miles/hour} \times (5280 \text{ ft/mile}) \times (\text{m}/3.28 \text{ ft}) \times (\text{hour}/60 \text{ min}) \times (\text{min}/60 \text{ s}) = 26.8 \text{ m/s}$$

Estimate cross-section area of car as $2 \text{ m} \times 3 \text{ m} = 6 \text{ m}^2$

$$F_D = 0.5 \times 0.2 \times 1.18 \text{ kg/m}^3 \times (26.8 \text{ m/s})^2 \times 6 \text{ m}^2 = 510 \text{ kg m/s}^2 = 510 \text{ Newton}$$

Power = $F_D \times v = 510 \text{ kg m/s}^2 \times 26.8 \text{ m/s} = 1.37 \times 10^4 \text{ kg m}^2/\text{s}^3 = 1.37 \times 10^4 \text{ W} = 18.3$ horsepower, which is reasonable

(b) Estimate the gas mileage of such a car. The heating value of gasoline is $4.3 \times 10^7 \text{ J/kg}$ and its density is 750 kg/m^3 .

$$\begin{aligned} \text{Fuel mass flow required} &= \text{power (Joules/s)} / \text{heating value (Joules/kg)} \\ &= 1.37 \times 10^4 \text{ kg m}^2/\text{s}^3 / 4.3 \times 10^7 \text{ J/kg} = 3.19 \times 10^{-4} \text{ kg/s} \end{aligned}$$

$$\begin{aligned} \text{Fuel volume flow required} &= \text{mass flow} / \text{density} \\ &= 3.19 \times 10^{-4} \text{ kg/s} / 750 \text{ kg/m}^3 = 4.25 \times 10^{-7} \text{ m}^3/\text{s} \times (3.281 \text{ ft/m})^3 \times 7.48 \text{ gal/ft}^3 \\ &= 1.12 \times 10^{-4} \text{ gal/sec} \end{aligned}$$