but with either definition, you can't have heat transfer being positive in one term and negative in the other term.

6. Only the first term on the right side of the equation is multiplied by the $e^{(-\alpha t/r^2)}$ factor, and thus will go to zero as t → ∞. So the other term would still be non-zero even when t → ∞, which doesn't make sense since the amount of heat transfer (q) has to go to zero as t → ∞. So probably both terms should be multiplied by the $e^{(-\alpha t/r^2)}$ factor.

Based on these considerations, a possibly correct formula, which would pass all of the smoke and function tests is

$$q = \left[kL(T_{bar} - T_\infty) + hr^2(T_{bar} - T_\infty) \right] e^{-\alpha t/r^2}$$

Actually even this is a bit odd since the first term (conduction heat transfer) is proportional to the length L but the second term (convection heat transfer) is independent of L … a still more likely formula would have both terms proportional to L, e.g.

$$q = \left[kL(T_{bar} - T_\infty) + hrL(T_{bar} - T_\infty) \right] e^{-\alpha t/r^2}$$

Example 3. Thermoelectric generator

The thermal efficiency (η) = (electrical power out) / (thermal power in) of a thermoelectric power generation device (used in outer planetary spacecraft (Figure 2), powered by heat generated from radioisotope decay, typically plutonium-238) is given by

$$\eta = \left(1 - \frac{T_L}{T_H}\right) \frac{\sqrt{1 + ZT_a} - 1}{\sqrt{1 + ZT_a} + T_L/T_H}; \; T_a \equiv \frac{T_L + T_H}{2} \qquad \text{(Equation 9)}$$

where T is the temperature, the subscripts L, H and a refer to cold-side (low temperature), hot-side (high temperature) and average respectively, and Z is the "thermoelectric figure of merit":

$$Z = \frac{S^2}{\rho k} \qquad \text{(Equation 10)}$$

where S is the *Seebeck coefficient* of material (units Volts/K, indicates how many volts are produced for each degree of temperature change across the material), ρ is the electrical resistivity (units ohm m) *(not to be confused with density!)* and k is the material's thermal conductivity (W/mK).

(a) show that the units are valid (passes smoke test)

Everything is obviously dimensionless except for ZT_a, which must itself be dimensionless so that I can add it to 1. Note